General Aptitude

(Numerical Ability)

&

(Verbal Ability)

For

All Streams

By

THE GATE ©
ACADEMY
A Forum of IIT / IISc Graduates

www.thegateacademy.com

✆ 080-40611000

Syllabus for General Aptitude

Verbal Ability: Grammar, Vocabulary, Coding-Decoding & Series, Directions, Blood Relations, Arrangements, Syllogism, Inference & Assumptions, Clocks and Puzzles

Numerical Ability: Fundamentals, Equations, Percentage, Averages, Ratio & Propotions, Mixture and Alligations, Data Interpretation & Data Suffiency, Time, Speed & Distance, Time & Work, Set Theory & Venn Diagrams, Progression, Functions & Graphs, Logarthims, Permutations and Combinations, Probability, Geometry & Mensuration

Previous Year GATE Papers and Analysis

GATE Papers with answer key

thegateacademy.com/gate-papers

Subject wise Weightage Analysis

thegateacademy.com/gate-syllabus

Contents Numerical Ability

Contents Verbal Ability

General Aptitude

Numerical Ability

By

THE GATE ©
ACADEMY
A Forum of IIT / IISc Graduates

www.thegateacademy.com

✆080-40611000

"In order to succeed, your desire for success should be greater than your fear of failure."

....Bill Cosby

CHAPTER 1

Numbers and Algebra

Numbers and Algebra are some of the most favourite topics of the examiners in any exam. Not only there is a large variety of questions that can be framed here, but also it provides the opportunity to test the problem solving skills of the students. We have segregated numbers into various subtopics which we'll be looking at one by one.

Numbers

Introduction

Natural Numbers: All positive integers are natural numbers.
Ex: 1, 2, 3, 4, 5,......

There are infinite natural numbers and '1' is the least natural number. Based on divisibility there would be two types of natural numbers. They are **Prime and Composite.**

Prime Number: A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.
Note:-Unity (i.e 1) is not a prime number.

Procedure to Check a Number is Prime or Not

1. Take the square root of the number.
2. Round off the square root to the next highest integer and call this number as z.
3. Check for divisibility of the number N by all prime numbers below z. If there is no prime number below the value of z which divides N then the number N will be prime.

Example: IS 241 is prime or not?
$\sqrt{241}$ lies between 15 and 16.Hence take the value of Z=16.
Prime numbers less than 16 are 2, 3, 5, 7, 11 and 13.
241 is not divisible by any of these. Hence we can conclude that 241 is a prime number.

Composite Numbers: The numbers which are not prime are known as composite numbers.
Note: 1 is neither prime nor composite

Co-Primes: Two numbers 'a' and 'b' are said to be co-primes, if their H.C.F is 1.
Example (2,3),(4,5),(7,9),(8,11).....

Tests for Divisibility

1. A number is divisible by 2, when its unit digit is even or 0.
2. A number is divisible by 3, when the sum of its digits is divisible by 3.
3. A number is divisible by 4 when the number formed by the last two digits on right hand side is divisible by 4, or if the last two digits are zeros.
4. A number is divisible by 5, when its unit digit is 5 or 0.
5. A number is divisible by 6, when it is divisible by 2 and 3 both.
6. **Divisibility test of 7**

   Method 1:

   If the digits a, b, c, d of a four-digit number abcd are such that $2b + 3c + d - a$ is divisible by 7, then the original number is divisible by 7

   e.g., $1981 = 2 \times 9 + 3 \times 8 + 1 - 1 = 42$ which is divisible by 7.

   Hence, 1981 is divisible by 7.

   Method 2:

   A number is divisible by 7 if the sum of the product of the digits of the number from left to right with $1, -2, -3, -1, 2, 3, \ldots$ successively is divisible by 7 or is 0.

   e.g., 392

   The required sum

   $= 3 \times 1 - 9 \times 2 - 3 \times 2 = -21$ which is divisible by 7. Hence, 392 is divisible by 7.

   Method 3:

   An integer I is divisible by 7, if the difference of the number of its thousands and the remainder of its division by thousand is divisible by 7.

   e.g., 439187

   Difference $= 439 - 187 = 252$ which is divisible by 7. Hence, 439187 is divisible by 7.

   Method 4:

   Any number is divisible by 7, if the number of tens added to five times the number of units is divisible by 7.

   e.g., 308

   Number of tens $= 30$

   The required sum $= 30 + 5 \times 8 = 70$ which is divisible by 7. Hence, 308 is divisible by 7.

   Method 5:

   Any number is divisible by 7, if the number of tens added to (-2) times the number of units is divisible by 7.

   e.g., 6727

   Number of tens $= 672$

   (-2) times the number of units $= -14$

   $672 - 14 = 658$

   Number of tens $= 65$

   (-2) times the number of units $= -16$

   $65 - 16 = 49$ which is divisible by 7. Hence, 6727 is divisible by 7.

7. A number is divisible by 8 when the number formed by the last three right hand digits is divisible by 8, or when the last three digits are zeros.
8. A number is divisible by 9, when the sum of its digits is divisible by 9.
9. A number is divisible by 10, when its unit's digit is 0.
10. A number is divisible by 11, when the absolute difference between the sum of the digits in the odd places and the sum of the digits in the even places is 0 or a multiple of 11.
    Note: When any number with an even number of digits is added to its reverse, the sum is always a multiple of 11.
11. A number is divisible by 12, when it is divisible by 3 and 4 both.
12. **Divisibility test of 13**

    Method 1:
    Any four digit number abcd is divisible by 13 if $a + 4b + 3c - d$ is divisible by 13.
    e.g., 9373
    $9 + 4 \times 3 + 3 \times 7 - 3 = 39$ which is divisible by 13.
    Hence, 9373 is divisible by 13.

    Method 2:
    A number is divisible by 13 if the sum of the product of the digits of the number from left to right with 1, 4, 3, −1, −4, −3, 1, 4, 3, … successively is divisible by 13 or is 0.
    e.g., 195
    The sum $1 \times 1 + 9 \times 4 + 5 \times 3 = 52$ which is divisible by 13.
    Hence, the number 195 is divisible by 13.

    Method 3:
    An integer I is divisible by 13, if the difference of the number of its thousands and the remainder of its division by thousand is divisible by 13.
    e.g., 160485
    Number of its thousands = 160
    Remainder of its division by 1000 = 485
    $160 - 485 = -325$
    which is divisible by 13.
    Hence, 160485 is divisible by 13.

    Method 4:
    Any number is divisible by 13, if the number of tens added to four times the number of units is divisible by 13.
    e.g., 6058
    Number of tens = 605
    4 times number of units = 32
    $605 + 32 = 637$
    which is divisible by 13.
    Hence, 6058 is divisible by 13.

13. A number is divisible by 15, when it is divisible by 3 and 5 both.

14. **Divisibility test of 17**

Method 1

A number is divisible by 17, if the number of tens added to 12 times the number of units is divisible by 17.
e.g., 153
Number of tens = 15
The required sum $= 15 + 12 \times 3 = 51$ which is divisible by 17
Hence, 153 is divisible by 17.

Method 2

A number is divisible by 17, if the number of tens added to (-5) times the number of units is divisible by 17.
In the same case as above, the required sum $= 15 + (-5) \times 3 = 0$ is divisible by 17.
Hence, 153 is divisible by 17.

15. A number is divisible by 19, if the number of tens is added to twice the number of units is divisible by 19.
e.g., 228
Number of tens = 22
The required sum $= 22 + 2 \times 8 = 38$ which is divisible by 19
Hence, 228 is divisible by 19.

16. A number is divisible by 25, when the number formed by the last two right hand digits is '00' or is divisible by 25.

17. A number is divisible by 29, if the number of tens added to thrice the number of units is divisible by 29.
e.g., 348
Number of tens = 34
The required sum $= 34 + 3 \times 8 = 58$ which is divisible by 29
Hence, 348 is divisible by 29.

18. A number is divisible by 125, when the number formed by the last three right hand digits is '000' or is divisible by 125.

Example: To find whether 467 is prime or not.

Solution: We find by trial that 467 is not divisible by any of the primes 2, 3, 5, 7, …… 19, 23. But on dividing by 23 we get a quotient less than 23, and we need not go further. For example, if 467 contained a prime factor beyond 23, the quotient obtained on dividing 467 by that prime factor would be less than 23, and must have been revealed as a factor by the former trials. Hence, 467 has no prime factor greater or less than 23; that is, it is a prime number.

In general, to find whether a number is prime or not, we need to check whether it is divisible by any of the primes up to the square root of that number.

The square of a composite number must contain the square of every factor of that number.

e.g., $6 = 2 \times 3 : 6^2 = 2 \times 3 \times 2 \times 3 = 2^2 \times 3^2$

$56 = 2^3 \times 7 : 56^2 = 2^3 \times 7 \times 2^3 \times 7 = 2^6 \times 7^2$

It will be noticed that each prime factor of the number is repeated an even number of times in the square of the number. Conversely, when a square of a number has been expressed in prime factors, its square root can be written down at once by simply the index of the power of each prime factor.

Even and Odd Numbers

Even Numbers: Numbers which are divisible by 2 are called even numbers. General notation is 2n, where n is an integer. **Example:** 2, 4, 6, 8,……….etc.

Odd Numbers: Numbers which are not divisible by 2 are called odd numbers. General notation is 2n+ 1 (or) 2n – 1, where n is an integer. **Example:** 3, 5, 7, 9……… etc.

Note: Let e and o represent even and odd numbers respectively,

(a) $e + e = e$
(b) $o + o = e$
(c) $e + o = o$
(d) $e \times e = e$
(e) $o \times o = o$
(f) $e \times o = e$
(g) $e^x = e$, where x is even or odd
(h) $o^x = o$, where x is even or odd

Example: If x is an even number and y is an odd number, then which of the following statements is false?

(i) $(x + x^x)(y + y^y)$ is even.
(ii) $(x + y) + (xy + y^x) + (x^y + x^x)$ is odd.
(iii) $x + y^x$ is odd.
(iv) $(x + y) + (xy + y) + x^y + y^x$ is odd.

Solution:

(i) $x + x^x$ = even + even = even; $y + y^y$ = odd + odd = even.
$(x + x^x)(y + y^y)$ is even. Hence, true.

(ii) $(x + y) + (xy + y^x) + (x^y + x^x)$ = odd + (even + odd) + (even + even)
= odd + odd + even = even. Hence, false

(iii) $x + y^x$ = even + odd = odd. Hence true.

(iv) $(x + y) + (xy + y) + x^y + y^x$ = odd + (even + odd) + even + odd
= odd + odd + even + odd
= even + odd = odd. Hence, true.

Example: A number P4571203R is divisible by 18. Which of the following values can P and R take?

(i) 1, 2
(ii) 2, 3
(iii) 6, 8
(iv) 3, 3